String action with multiplet of Θ-terms and the hidden Poincare symmetry

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Abstract

We study string action with multiplet of Θ-terms added, which turns out to be closely related with the bosonic sector of $D = 11$ superstring action [3,4]. Alternatively, the model can be considered as describing class of special solutions of the membrane. An appropriate set of variables is find, in which the light-cone quantization turns out to be possible. It is shown that anomaly terms in the algebra of the light-cone Poincare generators are absent for the case $D = 27$.

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1 Introduction

Construction of $D = 11$ Green-Schwarz type superstring action presents a nontrivial problem already at the classical level. The reason is that only for the dimensions $D = 3, 4, 6, 10$ the action is invariant under the local κ-symmetry (as well as under the global supersymmetry) [1]. Recently it was recognized [2-6] that the problem can be resolved if one introduces an additional vector variable n^N into the formulation. The corresponding $D = 11$ action (which incorporates $n^N(\tau, \sigma)$ as the dynamical variable) was suggested in [3]. Similarly to the Green-Schwarz construction, the action has κ-symmetry which allows one to remove half of fermionic coordinates and supply free dynamics for the physical variables as well as the discrete mass spectrum [3,4]. Moreover, n^N-independent part of spectrum (being classified with respect to $SO(1, 9)$ group) was identified with the type IIA superstring states. For the massless level classified with respect to $SO(1, 10)$ group one gets the supergravity

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multiplet in $D = 11$ [7-9]. Other states (presented on each mass level) may correspond to the states of the uncompactified M-theory limit [9,10]. Due to these properties one hopes that such a kind theory can be reasonable extension of the Green-Schwarz action to the case $D = 11$.

The aim of this work is to investigate some quantum properties of the bosonic toy model inspired by $D = 11$ superstring action (short version of the work is presented in [11]). The model is specified in Sec. 2 by mean of its own system of the Hamiltonian constraints in D-dimensional Minkowski space-time. The system contains all the necessary information for discussion of the light-cone quantization (remind also that any reparametrisation invariant free theory is determined unambiguously by given set of the constraints). It is demonstrated that the light-cone quantization is possible, which allows one to compute algebra of the light-cone Poincare generators. We show that anomaly terms in the algebra are absent for the case $D = 27$. Note that generalisation of the present analysis to the supersymmetric case is straightforward since fermionic sector of the superstring action do not involves of extra auxiliary fields.

Lagrangian formulation for the model is discussed in Sec. 3. We present two different Lagrangian actions, both of them reproduce the model under consideration in the Hamiltonian formulation. The first action has only $(D-1)$-dimensional manifest Poincare invariance and represents string with multiplet of Θ-terms added [9, 16]. The second action has D-dimensional manifest Poincare invariance and turns out to be closely related with the bosonic sector of $D = 11$ superstring considered in [3, 4].

Besides the string coordinates, D-dimensional action involves some auxiliary variables (in particular, the abovementioned vector $n^N(\tau, \sigma)$). In Sec. 4 we discuss a possibility that these variables (and the corresponding terms in the action) originate from the membrane action [1]. Namely, we select particular class of solutions of the membrane equations of motion, which preserve manifest $d = 2$ reparametrisation invariance. Our D-dimensional action has the same class as a general solution of equations of motion and thus can be considered as a theory which describe this particular sector of the membrane. Let us stress that contrary to the complete membrane theory [20, 21], the restricted version has discrete mass spectrum and definite critical dimension. In Conclusion we enumerate results of the work and discuss relation of the model considered

[1]Note that role of the auxiliary variables in the supersymmetric version is to supply the local κ-symmetry. The last was established in [3, 4] by using of the same $D = 11$ γ-matrix identity as for the supermembrane [18].

with the bosonic sector of $D = 11$ superstring action.

2 Light-cone quantization of the model and the critical dimension.

Besides the standard closed string coordinates $\tilde{x}^N$, $\tilde{p}^N$, $\tilde{\alpha}^N_n$, $\tilde{\bar{\alpha}}^N_n$, the model involve a pair of the (real) conjugated variables $\tilde{y}^N$, π^N, where π^N is zero mode of the abovementioned vector $n^N(\tau, \sigma)$. Our starting point is D-dimensional Virasoro constraints $(N = 0, 1, \ldots, D-1)$

$$L_n = \frac{1}{2}\sum_{\forall k} \tilde{\alpha}^N_{n-k}\tilde{\alpha}^N_k = 0, \quad \bar{L}_n = \frac{1}{2}\sum_{\forall k} \tilde{\bar{\alpha}}^N_{n-k}\tilde{\bar{\alpha}}^N_k = 0, \tag{1}$$

accompanied by the following second class system

$$\pi^N \tilde{\alpha}^N_n = 0, \qquad \pi^N \tilde{\bar{\alpha}}^N_n = 0, \qquad n \neq 0; \tag{2}$$

$$\pi^N \tilde{\alpha}^N_0 = 0, \qquad \pi^N \tilde{x}^N = 0, \tag{3}$$

which implies $\pi^N \pi^N \neq 0$. Below we will omit expressions for the left moving oscillators $\tilde{\bar{\alpha}}^N$. The cases of $SO(1, D-1)$ and $SO(2, D-2)$ group will be considered simultaneously: $\eta^{NM} = (\eta^{\mu\nu}, \eta^{D-1,D-1} \equiv \eta)$, $\eta = \pm 1$, $\eta^{\mu\nu} = (-, +, \ldots, +)$, $\mu, \nu = 0, 1, \ldots, D-2$. The parameter η is not fixed (except the restrictions which follow from the constraints) throughout the work, but is expected to be fixed in the supersymmetric version [4]. The string tension is chosen to be $T = \frac{1}{4\pi}$ such that $\tilde{\alpha}^N_0 = -\tilde{\bar{\alpha}}^N_0 = \tilde{p}^N$. Note that one more condition $\pi^2 = const$ can be added to the system (1)-(3) without spoiling of the subsequent analysis [11]. Spectrum is formed by action on the vacuum of oscillator modes only [6]. So, the sector $\tilde{y}^N, \pi^N$ can not produce extra negative norm states. D-dimensional Poincare generators are realized as

$$\mathbf{P}^N = -\tilde{p}^N, \qquad \mathbf{J}^{MN} = \tilde{x}^{[M}\tilde{p}^{N]} + iS^{MN} + i\bar{S}^{MN} + \tilde{y}^{[M}\pi^{N]},$$
$$S^{MN} \equiv \sum_{n=1}^{\infty} \frac{1}{n}\tilde{\alpha}^{[M}_{-n}\tilde{\alpha}^{N]}_n. \tag{4}$$

Below we present and discuss two possible interpretations for the system (1)-(3) in the Lagrangian framework. First, equivalent to those of (1)-(3) system can be reproduced starting from action of $(D-1)$-dimensional string with multiplet of D Θ-terms added [2].

[2]Note that string with one Θ-term is known to be equivalent to D-string (see [16,17] and references therein), where it can be easily taken into account in the path integral framework. It can be clue to understanding of n^N-dependent part of spectrum.

While the action has only manifest $(D-1)$-dimensional Poincare invariance, the correspondence means that it has also hidden D-dimensional Poincare symmetry. We show that it can be actually rewritten in a manifestly D-dimensional Poincare invariant form. It gives the second interpretation: the resulting action turns out to be closely related to the bosonic sector of $D=11$ superstring. Namely, the system (1), (2) can be obtained by means of partial fixation of gauge for the bosonic constraints presented in the theory [6]. As it was shown in [3,4], these constraints (and the corresponding terms in the action) are essential for establishing of the κ-symmetry.

Our aim now will be to perform light-cone quantization of the system (1)-(3). Then we show that anomaly terms in the light-cone Poincare algebra are absent for the critical dimension being $D=27$. Note that it is not surprising result since from Eq.(2) it follows that one component of each oscillator is nonphysical degree of freedom. So one expects that only the remaining $D-1$ components will give contribution into the anomaly terms, such that the condition of absence of the anomaly will be: $D-1=26$. We support this suggestion by direct calculations.

To quantize the theory we follow to the standard prescription [12,13]. The second class constraints (2), (3) can be taken into account by means of introduction of the corresponding Dirac bracket. For our basic variables the non zero brackets turn out to be

$$\begin{aligned}
\left\{\tilde{x}^N, \tilde{p}^M\right\} &= \Pi^{NM} \equiv \eta^{NM} - \frac{1}{\pi^2}\pi^N\pi^M, \\
\left\{\tilde{\alpha}^N_n, \tilde{\alpha}^M_k\right\} &= in\delta_{n+k,0}\Pi^{NM}, \\
\left\{\tilde{y}^N, \pi^M\right\} &= \eta^{NM}, \\
\left\{\tilde{y}^N, \tilde{y}^M\right\} = -\frac{1}{\pi^2}\tilde{x}^{[N}\tilde{p}^{M]} &- i\sum_{n=1}^{\infty}\frac{1}{n\pi^2}(\tilde{\alpha}^{[N}_{-n}\tilde{\alpha}^{M]}_n + \tilde{\bar{\alpha}}^{[N}_{-n}\tilde{\bar{\alpha}}^{M]}_n), \\
\left\{\tilde{x}^M, \tilde{y}^N\right\} = \frac{1}{\pi^2}\pi^M\tilde{x}^N, \qquad &\left\{\tilde{p}^M, \tilde{y}^N\right\} = \frac{1}{\pi^2}\pi^M\tilde{p}^N, \\
\left\{\tilde{\alpha}^M_n, \tilde{y}^N\right\} &= \frac{1}{\pi^2}\pi^M\tilde{\alpha}^N_n,
\end{aligned} \tag{5}$$

and the same expressions for the left moving oscillators $\tilde{\bar{\alpha}}^N_n$. Now Eqs.(2),(3) can be solved

$$\tilde{z}^{D-1} = -\frac{\eta}{\pi^{D-1}}\pi^\nu\tilde{z}^\nu, \tag{6}$$

where $\tilde{z} = (\tilde{x}, \tilde{p}, \tilde{\alpha}_n, \tilde{\bar{\alpha}}_n)$. Since brackets for the remaining variables $\tilde{x}^\nu, \tilde{p}^\nu, \tilde{\alpha}^\nu_n, \tilde{y}^N, \pi^N$ are rather complicated, it is convenient to simplify

them by means of an appropriate variable change. The change turns out to be (where $(\pi\tilde{x}) \equiv \pi^\nu \tilde{x}^\nu$ through this section)

$$\begin{aligned} x^\mu = \tilde{x}^\mu + c\pi^\mu(\pi\tilde{x}), \qquad p^\mu &= \tilde{p}^\mu + c\pi^\mu(\pi\tilde{p}), \\ \alpha_n^\mu &= \tilde{\alpha}_n^\mu + c\pi^\mu(\pi\tilde{\alpha}_n), \\ y^\mu &= \tilde{y}^\mu + c\left[(\pi\tilde{x})\tilde{p}^\mu - (\pi\tilde{p})\tilde{x}^\mu\right] + \\ ic\sum_{n=1}^{\infty}\left[\frac{1}{n}(\pi\tilde{\alpha}_{-n})\tilde{\alpha}_n^\mu + (n \leftrightarrow -n)\right] &+ (\tilde{\bar{\alpha}} - sector), \\ y^{D-1} &\equiv \tilde{y}^{D-1}. \end{aligned} \tag{7}$$

The factor c is any solution of the equation $\pi^2c^2+2c-\eta(\pi^{D-1})^{-2}=0$, thus

$$c = \frac{1}{\pi^2}\left[-1 \pm \frac{(\eta\pi^N\pi^N)^{\frac{1}{2}}}{\pi^{D-1}}\right]. \tag{8}$$

The new variables obey to the canonical brackets

$$\{x^\mu, p^\nu\} = \eta^{\mu\nu}, \quad \{y^N, \pi^M\} = \eta^{NM}, \quad \{\alpha_n^\mu, \alpha_k^\nu\} = in\eta^{\mu\nu}\delta_{n+k,0}. \tag{9}$$

Eq.(7) is invertible, an opposite change has the same form and can be obtained from Eq.(7) by means of substitution $z \leftrightarrow \tilde{z}$, $y \leftrightarrow \tilde{y}$, $c \mapsto \bar{c}$, where

$$\bar{c} = \frac{1}{\pi^2}\left[-1 \pm \pi^{D-1}(\eta\pi^N\pi^N)^{-\frac{1}{2}}\right]. \tag{10}$$

Note that a variable change which leads to Eq.(9) is not unique. For example (for the Dirac bracket which corresponds to Eq.(2)) the following simple change

$$\begin{aligned} \alpha_n^\mu = \tilde{\alpha}_n^\mu - \pi^\mu\frac{\tilde{\alpha}_n^{D-1}}{\pi^{D-1}}, \qquad \alpha_{-n}^\mu \equiv \tilde{\alpha}_{-n}^\mu, \\ y^N = \tilde{y}^N + i\sum_{n=1}^{\infty}\frac{1}{n\pi^{D-1}}(\tilde{\alpha}_{-n}^N\tilde{\alpha}_n^{D-1} + \tilde{\bar{\alpha}}_{-n}^N\tilde{\bar{\alpha}}_n^{D-1}), \end{aligned} \tag{11}$$

gives also the canonical brackets for the new variables. The problem is that the Virasoro constraints, being rewritten in terms of these variables, will contain products of α_n^- oscillators: $L_n \sim p^+\alpha_n^- + \frac{1}{2}(\pi^+)^2\sum_{k=0}^{n-1}\alpha_{n-k}^-\alpha_k^- + \ldots$. It do not allows one to resolve the constraints in the light-cone gauge. In contrast, our change (7) leads to the "linearised" form of the constraints. Namely, substitution of Eqs.(6), (7) into Eq.(1) gives the expressions

$$L_n = \frac{1}{2}\sum_{\forall k}\alpha_{n-k}^\mu\alpha_k^\mu = 0, \quad \bar{L}_n = \frac{1}{2}\sum_{\forall k}\bar{\alpha}_{n-k}^\mu\bar{\alpha}_k^\mu,$$

$$L_0+\bar{L}_0=(p^\mu)^2+\sum_{k=1}^{\infty}(\alpha^\mu_{-k}\alpha^\mu_k+\bar{\alpha}^\mu_{-k}\bar{\alpha}^\mu_k)=0, \qquad (12)$$

$$L_0-\bar{L}_0=\sum_{k=1}^{\infty}(\alpha^\mu_{-k}\alpha^\mu_k-\bar{\alpha}^\mu_{-k}\bar{\alpha}^\mu_k)=0, \qquad \mu=0,1,\ldots,D-2 \quad (13)$$

which contain the variables $p^\mu, \alpha^\mu_n, \bar{\alpha}^\mu_n$ only. Now the light-cone quantization can be carried out in the standard form [7,14,15]. One imposes the gauge $x^+=\alpha^+_n=\bar{\alpha}^+_n=0$, then the variables $p^-, \alpha^-_n, \bar{\alpha}^-_n$ can be expressed through the remaining (D-3)-dimensional oscillators $\alpha^i_n,\ \bar{\alpha}^i_n,\ i=1,2,\ldots,D-3$

$$p^- = \frac{1}{2p^+}(L^{tr}_0+\bar{L}^{tr}_0-a), \quad \alpha^-_n=\frac{1}{p^+}L^{tr}_n, \quad \bar{\alpha}^-_n=-\frac{1}{p^+}\bar{L}^{tr}_n,$$
$$L^{tr}_n=\frac{1}{2}\sum_{\forall k}\alpha^i_{n-k}\alpha^i_k, \qquad L^{tr}_0=\frac{1}{2}(p^i)^2+\sum_{k=1}^{\infty}\alpha^i_{-k}\alpha^i_k. \quad (14)$$

The oscillators are arranged in the normal order, the corresponding normal ordering constant a is included into the expression for p^-.

By using of Eqs.(4), (7), (14) one obtains the light-cone Poincare generators which can be presented as

$$\begin{aligned}
\mathbf{P}^\mu &= \mathbf{P}^\mu_{(D-1)}+\bar{c}\pi^\mu(\pi\mathbf{P}_{(D-1)}),\\
\mathbf{J}^{\mu\nu} &= \mathbf{J}^{\mu\nu}_{(D-1)}+y^{[\mu}\pi^{\nu]},\\
\mathbf{P}^{D-1} &= \pm\eta(\eta\pi^N\pi^N)^{-\frac{1}{2}}(\pi\mathbf{P}_{(D-1)}),\\
\mathbf{J}^{\mu D-1} &= c\pi^{D-1}\pi^\nu\mathbf{J}^{\nu\mu}_{(D-1)}+y^{[\mu}\pi^{D-1]}.
\end{aligned} \qquad (15)$$

The quantities $\mathbf{P}_{(D-1)}, \mathbf{J}_{(D-1)}$ coincide with the standard $(D-1)$-dimensional Poincare generators of the closed string

$$\mathbf{P}^\mu_{(D-1)}=-p^\mu, \qquad \mathbf{J}^{\mu\nu}_{(D-1)}=x^{[\mu}p^{\nu]}+iS^{\mu\nu}+i\bar{S}^{\mu\nu},$$
$$S^{\mu\nu}=\sum_{n=1}^{\infty}\frac{1}{n}\alpha^{[\mu}_{-n}\alpha^{\nu]}_n, \qquad (16)$$

where it is implied that Eq.(14) was substituted. Note that $-M^2=(\mathbf{P}^\mu)^2+\eta(\mathbf{P}^{D-1})^2\equiv(p^\mu)^2$ from which it follows that the last from Eq.(12) actually gives the mass formula. Thus, in terms of the new variables (7), D-dimensional Poincare generators of the theory are presented through the usual $(D-1)$-dimensional one. It makes analysis of the anomaly terms an easy task. By construction, commutators of the quantities (15) form D-dimensional Poincare algebra modulo to the terms which can arise in the process of reordering of oscillators to the normal form. The quantities (15) have the

following structure: $A(y,\pi)+B(\pi)C_{(D-1)}(x,p,\alpha,\bar\alpha)$, where $C_{(D-1)}$ represents the generators (16). Then structure of any commutator is

$$\left[A^1(y,\pi),A^2(y,\pi)\right]+\left[A(y,\pi),B(\pi)\right]C_{(D-1)}+ \\ B^1(\pi)B^2(\pi)\left[C^1_{(D-1)},C^2_{(D-1)}\right]. \tag{17}$$

The first two terms can not contain of ordering ambiguoutes. So the only source of the anomaly can be commutators of $(D-1)$-dimensional generators (16). The dangerous commutator is known to be $\left[\mathbf{J}^{i-}_{(D-1)},\mathbf{J}^{j-}_{(D-1}\right]$, which must be zero. Its manifest form is

$$[\mathbf{J}^{i-}_{(D-1)},\ \mathbf{J}^{j-}_{(D-1)}]=\frac{1}{(p^+)^2}\Big[(L^{tr}_0-\bar L^{tr}_0+a)S^{ij}-(L^{tr}_0-\bar L^{tr}_0-a)\bar S^{ij}+ \\ \sum_{n=1}^{\infty}[\frac{D-3}{12}(n-\frac{1}{n})-2n](\alpha^{[i}_{-n}\alpha^{j]}_n+\bar\alpha^{[i}_{-n}\bar\alpha^{j]}_n)\Big], \tag{18}$$

which is actually zero on the constraint surface (13) and under the conditions

$$D=27,\qquad a=2. \tag{19}$$

Note that in terms of the variables (7) the same critical dimension arises immediately in the old covariant quantization framework also, since the no-ghost theorem can be applied without modifications to Eqs.(12),(13).

3 Lagrangian formulation for the model.

We have established that the constraint system (1)-(3) presents example of a model with the critical dimension $D=27$. So, it is interesting to discuss Lagrangian formulation which reproduces this Hamiltonian system. It is convenient to start from the partially reduced formulation with variables (7), since in this case there is no of crossing terms among the string coordinates and the auxiliary ones (compare (12), (13) with (1)-(3). Then the theory is specified by the variable set $x^\nu(\tau,\sigma),\ p^\nu(\tau,\sigma),\ y^N,\ \pi^N$ and by the standard $(D-1)$-dimensional Virasoro constraints (12), (13). It prompts to consider action of $(D-1)$-dimensional string with multiplet of D Θ-terms added

$$S_{(D-1)}=\frac{1}{4\pi}\int d^2\sigma\left[\frac{-g^{ab}}{2\sqrt{-g}}\partial_a x^\nu\partial_b x^\nu-n^N\epsilon^{ab}\partial_a A^N_b\right], \tag{20}$$

where $\nu = 0, 1, \ldots, D-2$, $N = 0, 1, \ldots, D-1$, $\epsilon^{ab} = -\epsilon^{ba}$, $\epsilon^{01} = -1$. All the variables obey to the periodic boundary conditions. The Lagrangian multiplier $A^N_a(\tau, \sigma)$ supplies $n^N(\tau, \sigma) = \pi^N = const$ on-shell (alternatively, $U(1)^D$ gauge invariance can be used to remove all modes of A^N_a, n^N except the zero one: $A^N_0 = 0$, $A^N_1(\tau, \sigma) = y^N + \pi^N \tau$, $n^N(\tau, \sigma) = \pi^N$). From this it follows that the action (20) actually leads to the desired picture in the Hamiltonian formalism. Being manifestly Poincare invariant in $(D-1)$ dimensions only, the action possess hidden D-dimensional Poincare symmetry, as it is clear from Eq.(15) (then A^N_a, n^N are considered as D-dimensional Lorentz vectors). So one expects that it can be rewritten in a manifestly D-dimensional Poincare invariant form. The relevant action is

$$S_D = \frac{1}{4\pi} \int d^2\sigma \left[\frac{-g^{ab}}{2\sqrt{-g}} D_a x^N D_b x^N - n^N \epsilon^{ab} \partial_a A^N_b \right], \qquad (21)$$

where $D_a x^N \equiv \partial_a x^N - \xi_a n^N$, and $\xi_a(\tau, \sigma)$ is one more auxiliary field. Local symmetries of the theory are $d = 2$ reparametrizations, Weyl symmetry and the following transformations with the parameters γ, α^N

$$\delta x^N = \gamma n^N, \qquad \delta \xi_a = \partial_a \gamma, \qquad \delta A^N_a = \gamma \frac{\epsilon_{ab} g^{bc}}{\sqrt{-g}} D_c x^N; \qquad (22)$$

$$\delta A^N_a = \partial_a \alpha^N. \qquad (23)$$

As it should be, total number of the parameters coincide with the number of primary first class constraints (26). Let us demonstrate that the action (21) reproduces the equations (1)-(3) in the Hamiltonian formulation. By direct application of the Dirac algorithm one finds the Hamiltonian

$$\begin{aligned} H = \int d\sigma \Big\{ -\frac{N}{2} \left[\frac{1}{4\pi} p^2 + 4\pi (\partial_1 x^N - \xi_1 n^N)^2 \right] \\ -N_1 p^N (\partial_1 x^N - \xi_1 n^N) + \xi_0 (np) + 4\pi (n \partial_1 A_0) + + \lambda^{ab}_{(g)} p_{(g)ab} + \\ + \lambda_{(\xi)a} p^a_{(\xi)} + \lambda^N_{(A)0} p^{0N}_{(A)} + \lambda^N_{(A)1} (p^{1N}_{(A)} - 4\pi n^N) + \lambda^N_{(n)} p^N_{(n)} \Big\}, \end{aligned} \qquad (24)$$

where $p_{(q)}$ is momenta conjugate to the variable q, and $\lambda_{(q)}$ are Lagrangian multipliers for the primary constraints. It was denoted also $N \equiv \frac{\sqrt{-g}}{g^{00}}$, $N_1 \equiv \frac{g^{01}}{g^{00}}$. After determining of the secondary constraints (there are no of tertiary constraints in the problem), the complete constraint system can be presented in the form

$$n^N = \frac{1}{4\pi} p^{1N}_{(A)}, \qquad p^N_{(n)} = 0,$$

$$\xi_1 = 4\pi \frac{(p^1_{(A)}\partial_1 x)}{(p^1_{(A)})^2}, \qquad \pi^1_{(\xi)} = 0; \tag{25}$$

$$p_{(g)ab} = 0, \qquad p^0_{(\xi)} = 0, \qquad p^{0N}_{(A)} = 0; \tag{26}$$

$$\partial_1 p^{1N}_{(A)} = 0; \tag{27}$$

$$\left[\frac{1}{4\pi}p^N \pm (\partial_1 x^N - \frac{(p^1_{(A)}\partial_1 x)}{(p^1_{(A)})^2} p^{1N}_{(A)})\right]^2 = 0, \quad p^{1N}_{(A)} p^N = 0. \tag{28}$$

Note that rank of the matrix formed by the Poisson brackets of the constraints depends on the value of π^2. So, the sectors $\pi^2 \neq 0$ and $\pi^2 = 0$ correspond to essentially different theories. In particular, in the case $\pi^2 = 0$ one finds the first class constraints $\pi^1_\xi = 0$, $(p^1_{(A)}\partial_1 x) = 0$ instead of the second class pair from Eq.(25). We restrict our consideration to the sector $\pi^2 \neq 0$. Then the constraints (25) are of second class, while the remaining ones are of first class. An appropriate gauge for Eq.(26) is

$$g^{ab} = \eta^{ab}, \quad \xi_0 = 0, \quad A^N_0 = -\int_0^\sigma d\sigma' \left[\xi_1 D_1 x^N - p^{1N}_{(A)}\right]. \tag{29}$$

Now Eqs.(25), (26), (29) can be taken into account by means of introduction of the Dirac bracket. Then the variables g^{ab}, $p_{(g)ab}$, n^N, $p^N_{(n)}$, ξ_a, $p^a_{(\xi)}$, A^N_0, $p^{0N}_{(A)}$ can be omitted from consideration. The Dirac bracket for the remaining variables A^N_1, $p^{1N}_{(A)}$, x^N, p^N coincide with the Poisson one. In the gauge chosen equations of motion for the sector A^N_1, $p^{1N}_{(A)}$ turn out to be linear

$$\partial_0 A^N_1 = p^{1N}_{(A)}, \qquad \partial_0 p^{1N}_{(A)} = 0. \tag{30}$$

An appropriate gauge for the constraint (27) of this sector is [5, 6] $\partial_1 A^N_1 = 0$. The only remaining degrees of freedom in this gauge are zero modes

$$A^N_1(\tau,\sigma) = y^N + \pi^N \tau, \qquad p^{1N}_{(A)}(\tau,\sigma) = \pi^N = const. \tag{31}$$

Dynamics in the sector x^N, p^N is governed now by the equations

$$\partial_0 x^N = \frac{1}{4\pi}p^N, \qquad \partial_0 p^N = 4\pi \Pi^N{}_M \partial_1\partial_1 x^M, \tag{32}$$

where $\Pi^N{}_M = \delta^N_M - \frac{1}{\pi^2}\pi^N\pi_M$. The remaining constraints acquire the form

$$\left[\frac{1}{4\pi}p^N \pm \Pi^N{}_M\partial_1 x^M\right]^2 = 0, \qquad \pi^N p^N = 0. \tag{33}$$

In the gauge

$$\pi^N x^N = 0, \tag{34}$$

for the last constraint, Eq.(32) reduce to those of the usual string, with the well-known solution in terms of the oscillator variables. Being rewritten in these terms, Eqs.(33), (34) coincide with Eqs.(1)-(3), as it was stated above.

Thus, it was established canonical equivalence of the actions (20) and (21) -they have the same physical sector. In particular, while the action (20) has only manifest $(D-1)$-dimensional Poincare invariance, it possess also hidden D-dimensional Poincare symmetry, the last is given by Eqs.(15), (16).

4 The model as a special sector of the membrane.

Particular form of the action (21) was guessed above from the requirement that it reproduces the desired constraint system (1)-(3) in the partially fixed gauge. In this section we discuss a possibility to obtain this model starting from the membrane action. Membrane equations of motion are intrinsically non-linear, which do not allows one to obtain their general solution. Some special solutions were considered in the literature (see [18, 22] and references therein). In particular, there are solutions which correspond to the massive particle and to the string [18]. Semiclassical quantization of the membrane was investigated on the ground of the spherical solution [23] and of the toroidal one [19]. Here we select one more class which turns out to be useful in the present context. We look for solutions with ansatz for the membrane coordinate $x^N(\tau,\sigma,\rho)$ chosen in special form. After substitution of the ansatz into the membrane equations of motion they acquire the form (32), (33). So the action (21) can be considered as describing this particular sector of the membrane theory.

First note that the action (21) can be obtained from the membrane action [19]

$$S = \frac{1}{4\pi}\int d^3\sigma \frac{1}{2\sqrt{-\gamma}}\left[-\gamma^{AB}\partial_A x^N \partial_B x^N + 1\right], \tag{35}$$

by means of the following formal trick. Consider $d=2$ reparametrisation invariant truncation of the metric ($A=(a,2)$)

$$\gamma^{ab} = g^{ab}(\tau,\sigma), \quad \gamma^{a2} = -g^{ab}\xi_b(\tau,\sigma), \quad \gamma^{22} = 1 + g^{ab}\xi_a\xi_b;$$
$$g^{ab}g_{bc} = \delta^a_c, \qquad \det\gamma^{AB} = \det g^{ab}; \qquad (36)$$

and of the coordinate (cylindrical membrane is considered)

$$x^N(\tau,\sigma,\rho) = \tilde{x}^N(\tau,\sigma) + \frac{\pi^N}{\sqrt{\pi^2}}\rho, \qquad (37)$$

where $\pi^N = const$. Substitution of Eqs.(36), (37) into Eq.(35) gives the expression

$$S = \frac{1}{4\pi}\int d^2\sigma \frac{-g^{ab}}{2\sqrt{-g}}\left(\partial_a\tilde{x}^N - \xi_a\pi^N\right)^2. \qquad (38)$$

Further, to avoid appearance of the fixed vector π^N in the formulation, one introduces the dynamical variable $\pi^N \longrightarrow n^N(\tau,\sigma)$. The condition $\partial_a n^N = 0$ can be incorporated into Eq.(38) by means of the Lagrangian multiplier term as follows

$$S_D = \frac{1}{4\pi}\int d^2\sigma\left[\frac{-g^{ab}}{2\sqrt{-g}}(\partial_a\tilde{x}^N - \xi_a n^N)^2 - \epsilon^{ab}A^N_a\partial_b n^N\right]. \qquad (39)$$

From equations of motion $\frac{\delta S}{\delta A} = 0$ one has $n^N(\tau,\sigma) = \pi^N = const$, as it is desired. The resulting action (39) coincides with Eq.(21).

This trick will be legitimate if the truncation (36), (37) is consistent with the membrane dynamics. This fact can be easily demonstrated in the Hamiltonian formulation [22, 19]. Actually, after partial fixation of gauge, the membrane dynamics is governed by the equations of motion ($\partial_A = (\partial_0, \partial_i)$)

$$\partial_0 x^N = \frac{1}{4\pi}p^N,$$
$$\partial_0 p^N = 4\pi\partial_1\left[(\partial_2x\partial_2x)\partial_1x^N - (\partial_1x\partial_2x)\partial_2x^N\right]$$
$$+4\pi\partial_2\left[(\partial_1x\partial_1x)\partial_2x^N - (\partial_1x\partial_2x)\partial_1x^N\right], \qquad (40)$$

and by the constraints

$$(p\partial_i x) = 0, \qquad (4\pi)^{-2}p^2 + \det(\partial_i x\partial_j x) = 0. \qquad (41)$$

Let us look for solutions with the ansatz (37). Substitution into Eqs.(40), (41) gives the equations [3]

$$\partial_0\tilde{x}^N = \frac{1}{4\pi}p^N, \qquad \partial_0 p^N = 4\pi\partial_1\left[\partial_1\tilde{x}^N - \frac{(\pi\partial_1\tilde{x})}{\pi^2}\pi^N\right],$$

[3] It is interesting to note that the ansatz $x^N(\tau,\sigma,\rho) = \tilde{x}^N(\tau,\sigma)$ gives one more class of the collapsed [18] solutions. Namely, substitution into Eqs.(40), (41) leads to the tensionless string dynamics [24-26].

$$(\pi p) = 0, \quad (p\partial_1\tilde{x}) = 0, \quad (4\pi)^{-2}p^2 + \left(\partial_1\tilde{x}^N - \frac{(\pi\partial_1\tilde{x})}{\pi^2}\pi^N\right)^2 = 0. \quad (42)$$

The last constraint implies, in particular, $p^N(\tau,\sigma,\rho) = \tilde{p}^N(\tau,\sigma)$. The resulting equations (42) are equivalent to the system (32), (33), the last was obtained from the action (21). General solution of the system was discussed in the previous section, which confirm consistency of the trick.

Note that truncation of the type (36), (37) can be applied to the supermembrane action as well. From the previous results one expects that the resulting supersymmetric theory has critical dimension $D = 11$.

5 Conclusion

In this work we have presented example of the bosonic string-type model which can be quantized in the light-cone gauge and leads to the critical dimension $D = 27$. Two canonically equivalent Lagrangian actions for the model were discussed, see Eq.(20) and Eq.(21), with manifest Poincare symmetry in $(D-1)$ and D dimensions correspondingly. There is analogy between the action (21) and D-string which can be clue for understanding of n^N-dependent part of spectrum. It was demonstrated that D-dimensional action can be considered as a theory which describe some class of special solutions of the membrane equations of motion. One expects that the truncation used can be equally applied to the supermembrane action, which would give supersymmetric version for the model considered.

Note also that analysis of spectrum in the light-cone gauge is expected to be more complicated as compare with the standard case. In the gauge considered the manifest symmetry is $SO(D-3)$ while the massive states should fall into representations of the little group $SO(D-1)$. Similar situation arise for $D = 11$ membrane [18,19] and was analyzed in [8]. It was demonstrated that $SO(8)$ multiplets of the first massive level for the toroidal supermembrane actually fall into representations of $SO(10)$ group. We hope that the analogous consideration is applicable for the present case as well.

To conclude, let us comment on relation between (21) and the bosonic sector of $D = 11$ superstring [3,4]. The only difference is appearance of the constraint $\pi^N\partial_1 x^N = 0$ instead of the last constraint from Eq.(33), which means that Eq.(3) is absent. One expects that for an appropriately chosen variables all the previous analysis can be repeated for this case also. The important point is that zero

string modes are not restricted, so quantum states of the theory (in particular, fields of the low-energy effective action) will be functions of all the momentum components p^N, which can simplify analysis of the state spectrum. Fermionic sector of $D = 11$ superstring action do not involves of extra auxiliary fields and consist of $D = 11$ Majorana spinor only. The last can be decomposed on a pair of the Majorana - Weyl spinors of an opposite chirality with respect to $SO(1, 9)$ group. From this fact and from the result $D = 27$ for the bosonic sector one expects that the critical dimension for the superstring presented in [3,4] is $D = 11$.

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